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3D GEOMETRIC MODELING

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Abstract. This article addresses an application of computer modeling in teaching and learning geometry. Our aim is to increase the interest of students in studying of geometry at secondary schools and colleges. One possible approach of improvement studying geometry is the integration of the computer software in the teaching process. The main field of our interest is the study of classical geometry and descriptive geometry – geometric constructions, projections, geometry of curves and surfaces. In this article we focus on studying geometry of surfaces and three dimensional geometry and we show some examples of 3D modeling. The explanation of some three dimensional geometric problem can be very difficult so 3D modeling software is very useful for understanding. We use computer software for visualization, for the proving of geometric problems or for demonstration of the application of geometry in practice. We show the advantages of 3D modeling software based on examples from the research achieved during my PhD studies. Nowadays we have a great collection of the outputs which are suitable aid and can be used in some publications and also for home schooling and e-learning not only for our students; see http://www.surynkova.info (still under the construction).

INTRODUCTION AND MOTIVATION

Geometry, the study of properties and relations of geometric figures, is an important and essential branch of mathematics. Geometry can be conceived as an independent discipline with many branches – Euclidean geometry, differential geometry, algebraic geometry, topology, no-Euclidian geometry, computational geometry, applied geometry and so on. Geometry is useful for learning other branches of mathematics and it can also be used in a wide range of scientific and technical disciplines. Some scientific branches require direct knowledge of geometry.

The main field of our interest is the study of classical geometry, descriptive geometry and computational geometry. Descriptive geometry is a branch of geometry which allows the representation of three dimensional objects in two dimensions. Generally this geometry studies geometric constructions in the plane and in the space, projections, geometry of curves and surfaces. Descriptive geometry which studies the properties of projections has become the language of designers, engineers and architects. Classical geometry is geometry of the Euclidean plane and space. Computational geometry is a branch of computer science devoted to the study of geometric algorithms for various practical applications. The subject and scope of my PhD studies are mainly from the field of the computational geometry. It is a great advantage to have knowledge of classical geometry because I use it in my research all the time. In this article our aim is to show practical uses of geometry. Mainly we will focus on examples of using geometry of surfaces, three dimensional geometry and geometric algorithms in computational geometry which I use in my scientific research.

Geometry is important for everyone, not only for technicians, designers, architects, builders or civil engineers. We all need good visual imagination in our everyday life as well. The two and three dimensional shapes which surround us are originated in geometry. The world we live in is influenced by geometry. If we know how to understand and apply the relationship between shapes and sizes we can use it more efficiently. Some people think in images and shapes so they need the understanding of geometry to be able to do that.

Without the use of geometry the great works of artists, painters and builders would only have stayed in ideas and dreams.

According to (Hilbert, 1999) the study of geometry develops logical reasoning and deductive thinking which helps us expand both mentally and mathematically. If we learn to use geometry we also learn to think logically. It’s very important in everyday life – many difficult problems can be erased and the simple solutions can be found. Students can often solve problems from other fields more easily if they represent the problems geometrically.

The rest of this paper is organized as follows: the next section “The study of geometry” is devoted to current problems of the unpopularity and the difficulty of studying geometry. “How to increase the interest of students in studying classical geometry; use of computers in the teaching process”
is the main subject of the following section. In the section “The examples of 3D modeling” we will demonstrate the advantages of modeling computer systems especially Rhinoceros on concrete examples from the field of descriptive geometry and geometry of surfaces. The models which illustrate examples of surfaces in building practice, architecture and industrial design will be shown. Finally in the section “Geometry in digital surface reconstruction from point clouds” we will explain some results of my research on PhD studies. Geometric algorithms and mathematical methods which I use will be exemplified in 3D modeling computer software. In conclusion we will discuss the advantages of geometric modeling systems and the responses from our undergraduate students.

THE STUDY OF GEOMETRY

The study of geometry can be very difficult. This branch of mathematics isn’t popular among students; see for example (Schwartz, 2008a; Schwartz, 2008b). Drawings (the results of geometric projections of some three dimensional object) are sometimes very difficult to understand. For that reason, geometric problems must be provided with clear examples. Intuitive understanding plays a major role in geometry. With the aid of spatial imagination we can illuminate the problems of geometry. In many cases, it is possible to show the geometric outline of the methods of investigation and proof without entering needlessly into details. The problem can be more understandable without strict definitions and actual calculations. Such intuition has a great value not only for researchers, but also for anyone who wishes to study and appreciate the results of research in geometry. Of course, if we understand the main principles of a problem then we can use formal definitions.

The current predominant view among students and the general public is that classical geometry is not important and useful. Drawings of classical geometry can be replaced by the outputs of modern computer software. Of course, computers can help us to solve geometric problems and to increase the efficiency of our work but we still have to know the basic principles and rules in geometry. We want to demonstrate that good knowledge of geometry can be very useful in scientific research in the area of practice of classical and applied geometry.

Is it possible to learn geometry? Yes, but it would be easier for students if they had encountered classical geometry, constructions and geometric proofs earlier. Sometimes students of technical specializations experience geometry only at college. That is too late. We work mainly with undergraduate students, so what can be done to make college geometry more comprehensible? How to increase the interest of students in studying classical geometry at secondary schools and colleges? This is the main subject of this article.

HOW TO INCREASE THE INTEREST IN STUDYING GEOMETRY; USE OF COMPUTERS IN THE TEACHING PROCESS

Our aim is to increase the interest of students in studying classical geometry at secondary schools and colleges. One possible approach of improving studying geometry is the integration of modern mathematical and modeling computer software in the teaching and learning process. This way seems to be interesting, attractive and motivational for students. Indeed the usage of computers in education is comparatively new and very current. Computers influence our everyday life including geometry. We have to follow the general trend.

Currently, computer-aided design (CAD) is commonly used in the process of design, design documentation, construction and manufacturing processes. There exist a wide range of software and environments which provide the user input tools for modeling, drawing, documentation and design process. These software and environments can be used to design curves and geometric objects in the plane and curves, surfaces and solids in the space. According to the applications more than just shapes can be affected in these software. In modern modeling software, we can also work with rotations and other transformations; we can change the view of a designed object. Some software provides dynamic modeling. These tools can help for better understanding the geometric situations mainly in the space. We can see spatial geometric objects from another view so it can be clearer how it looks like. Technical and engineering drawings must contain material information and the methods of construction. Computer-aided design is used in numerous fields: industry, engineering, science and many others. The particular use of computer varies according to the profession of the user and the type of software.

These modern methods which are widespread in various branches can be useful in the teaching process, too. We should help students to improve their skills for their future employment. We do not advocate the usage of computers at any price. If we use computer in the teaching and learning process, we still put emphasis on the understanding of the principles used in geometry. Sometimes it is of significant importance to work without technical support and use only our own mind. Software can work
automatically but this is not desirable in the teaching and learning of geometry. Good geometric imagination and perception is very important for understanding constructions in geometry. It is not possible to memorize the constructions; we have to understand geometric problems.

I have experiences teaching classical, descriptive, kinematic, and computational geometry at universities such as Charles University in Prague at the Faculty of Mathematics and Physics and Czech Technical University in Prague at the Faculty of Architecture. College geometry is very difficult for many students. It is necessary to motivate them and to arouse their interest in geometry. As was mentioned above, it is necessary to improve the teaching of geometry at elementary schools and at secondary schools.

In my lessons I use computer software for visualization, for the proving of geometric problems in the plane and in the space, for the demonstration of the application of geometry in practice, for the creations of study materials for home schooling and e-learning or for the transformation geometric problems into algebraic form and the other way around. I work mainly with Rhinoceros - NURBS modeling for Windows (Rhino), Cabri II Plus, Cabri 3D, MATLAB, Maple and GeoGebra. Use by teachers and students is always free of charge, it is the great advantage of GeoGebra. Consequently it can be used by students for home schooling and e-learning. Other software is available for teachers and students at our school. Of course, there exist alternative software which are free of charge. Nowadays we have extensive database of geometric tasks, images and 3D models – the outputs of these software; see http://www.surynkova.info.

3D GEOMETRIC MODELING

I use mathematical and 3D modeling computer software for creation of geometric constructions step by step which can help my students understand the problem in intuitive and natural way. Moreover I show special constructions applied in descriptive geometry and due to included functions and tools in these software students can discover proofs more easily. In this contribution we will demonstrate the advantages of the use of geometric modeling systems on examples from the field of descriptive geometry and computational geometry. Classical geometry is very useful branch of mathematics in practical applications. Geometry of surfaces is very important in building practice and architecture. The models which illustrate examples of the usage of special surfaces in architecture and technical design will be shown.

Geometry in the Space

How we said the study of geometry, especially descriptive geometry, can be very difficult. Mainly this geometry studies properties of projections of three dimensional objects in two dimensions. If we work only with the two dimensional representation of some spatial geometric object or situation, the shape, size and another characteristics of this object or situation can be problematic to imagine and understand. At this moment 3D modeling computer software seems to be very useful. See figure 1, there is the drawing of the shadow of the sphere in linear perspective, the directional light is defined by the arrow, we work only with the results of geometric projection in two dimensions. This task is not easy to solve and to create hand-drawn picture is also difficult. Next figure 2 shows the same task in 3D modeling computer software – a) front view, b) top view, c) situation in the space, d) situation in the space with the determination of linear perspective (*S* is the center of the projection and is the projection plane) and the projected image. In a), b), d) there are also illustrated imaginary cones of perspective sight. Figure 3 shows the rendered image of the same spatial situation. If we work with 3D modeling software, we can change the view of a designed object and see spatial geometric objects from another view so it can be clearer how it looks like. In this example we created in the 3D modeling software also the determination of linear perspective and demonstrated the main principles of this projection. Classical and descriptive geometry is due to suitable computer software more understandable and geometry in general becomes modern discipline.

These pictures were made in Rhinoceros - NURBS modeling for Windows. Rhinoceros (Rhino) is commercial NURBS-based 3D modeling tool. This software is commonly used in the process of design, design documentation or construction. Rhinoceros has many tools and functions for graphics designers. Rhinoceros can create, edit, analyze, document, render, animate NURBS curves, surfaces, and solids . Of course, there exist alternative software which are free of charge. We use this software because it is available for students and teachers at our school.

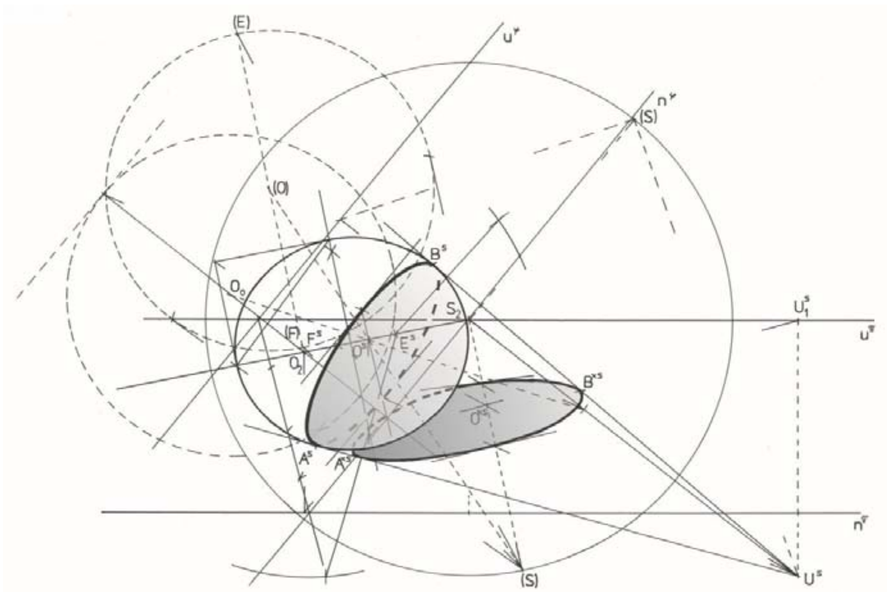


Figure 1: The shadow of the sphere in linear perspective. The situation in two dimension obtained by projection of the sphere into the plane, the picture is hand-made.

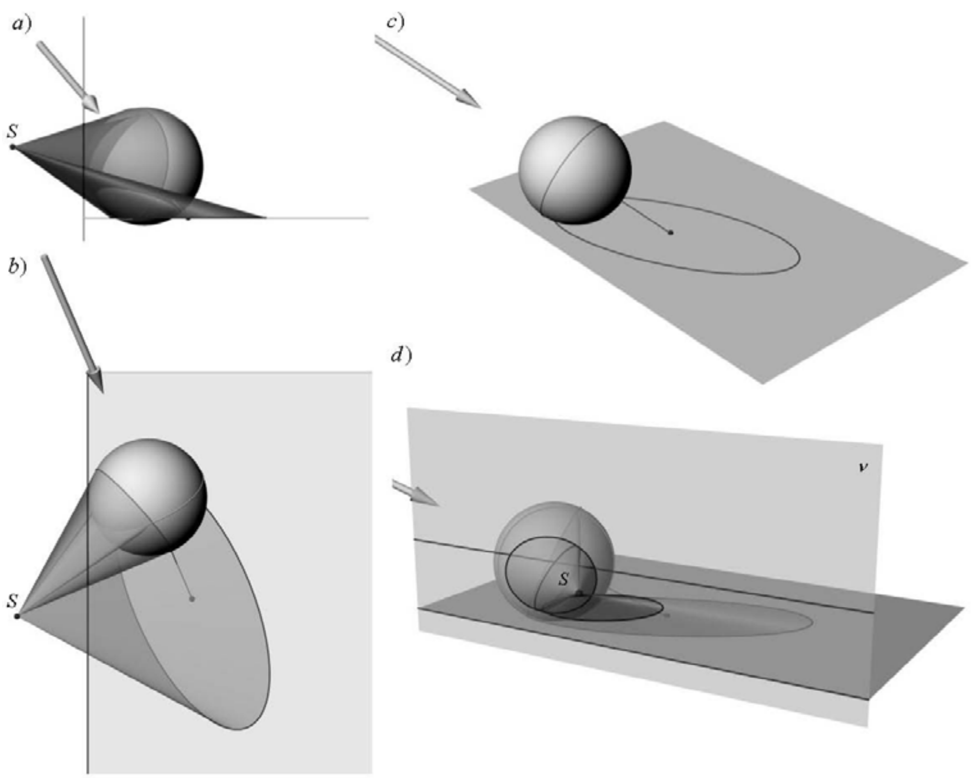


Figure 2: The same example of the shadow of the sphere in linear perspective in 3D modeling software Rhinoceros – a) front view, b) top view, c) situation in the space, d) situation in the space with the determination of linear perspective (S is the center of the projection and ν is the projection plane) and the projected image. In a), b), d) there are also illustrated imaginary cones of perspective sight. In every picture the arrows define the directional light.

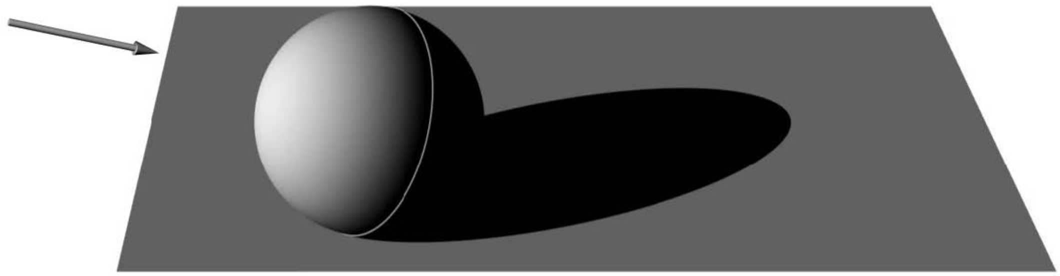


Figure 3: The same example of the shadow of the sphere in linear perspective in 3D modeling software Rhinoceros – rendered view. In this software we can change the view of a designed object. These tools can help for better understanding the geometric situations mainly in the space. We can see spatial geometric objects from another view so it can be clearer how it looks like.

If we want to work with similar computer modeling software (it is not necessary to work just with Rhinoceros), we have to know geometric principles not only the special tools and functions which that software provides. On one hand these software are useful aid in the popularization of geometry in general but on the other hand we need to understand geometry if we work with them. Mainly mathematical and modeling computer software can motivate our students to discover the beauty of geometry. Of course, then students have to learn geometry in classical way. Finally students can create the outputs with some software themselves and apply their knowledge gained from studying. The construction of the models is not just a computer amusement. See the figure 4 – next demonstration of the usage of Rhinoceros. There is typical example from descriptive geometry, the intersection of two surfaces of revolution. Again this task can be difficult to solve only in two dimensional projections.

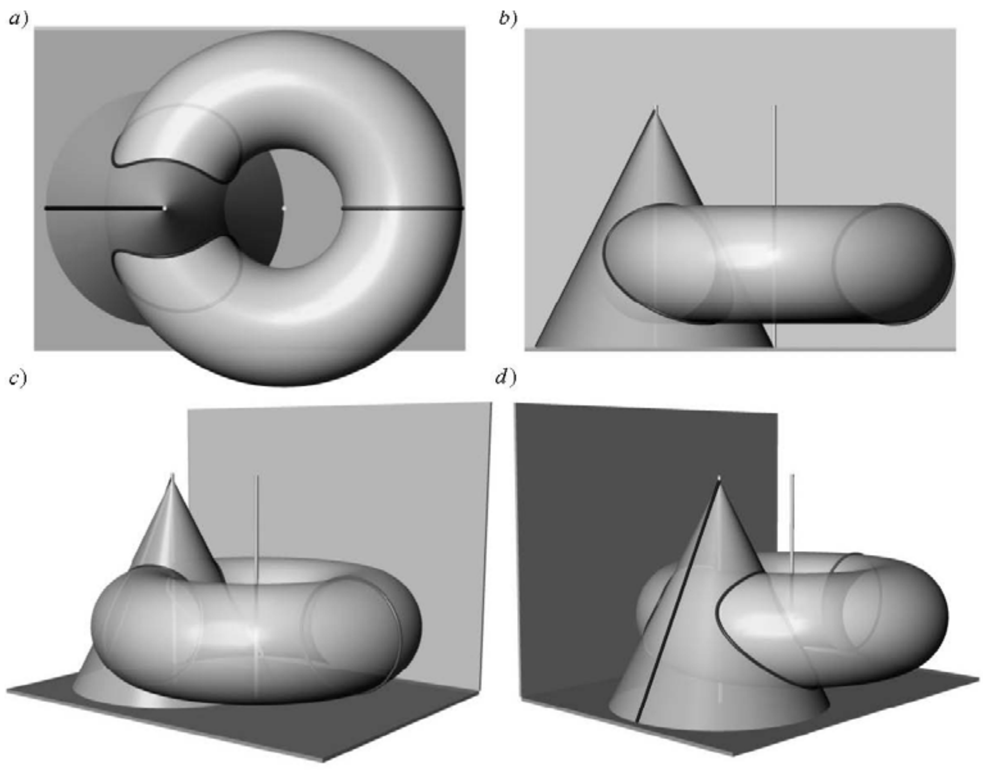


Figure 4: Intersection curves of two surfaces of revolution – a) top view, b) front view, c) and d) two perspective projections of intersection curves of two surfaces of revolution.

Geometry of Surfaces

The next important branch of descriptive geometry is the study of surfaces. Surfaces arise in a variety of applications, including art, architecture, and design. Traditional surface classes are largely based on a simple kinematic generation. They are swept by a profile curve undergoing a smooth motion. Descriptive geometry studies the determinations and properties of surfaces. A good knowledge of the basic concepts in connection with surfaces is essential for understanding. 3D modeling computer software is very good helper again. Now we can also use the animation for illustration the generation of surfaces.

Figure 5 shows the generation of surface of revolution, figure 6 demonstrates the generation of helical surface and figure 7 illustrates the generation of ruled surfaces. Theory of these surfaces can be found in (Pottman et al., 2007; Farin, 2002; Gerald et al., 2002).

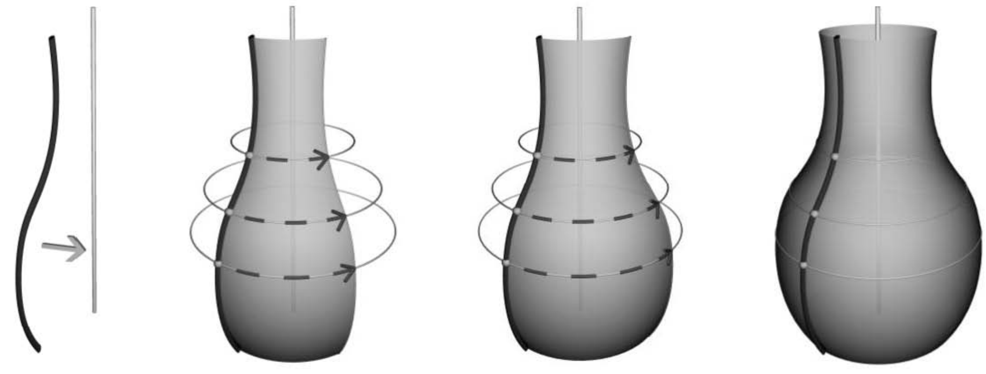


Figure 5: Illustration of the generation of surface of revolution by rotating a curve about an axis. Every point of the curve describes a circle with the center on the axis.

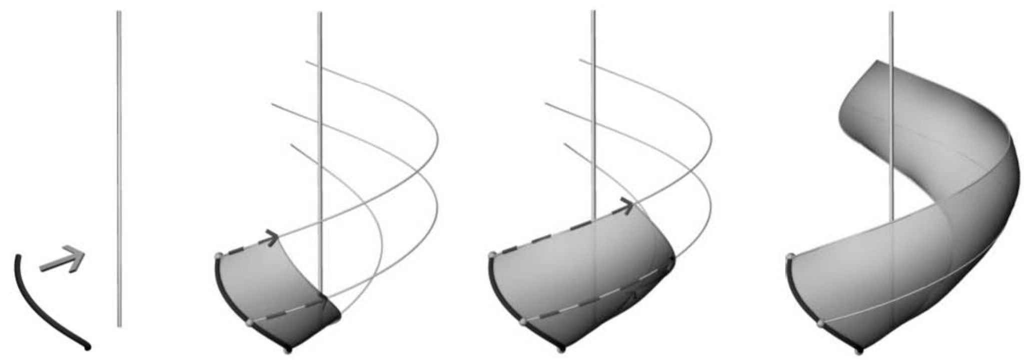


Figure 6: Illustration of the generation of helical surfaces by helical motion a curve about an axis. Every point of the curve describes a helix with the same axis as the axis of surface.

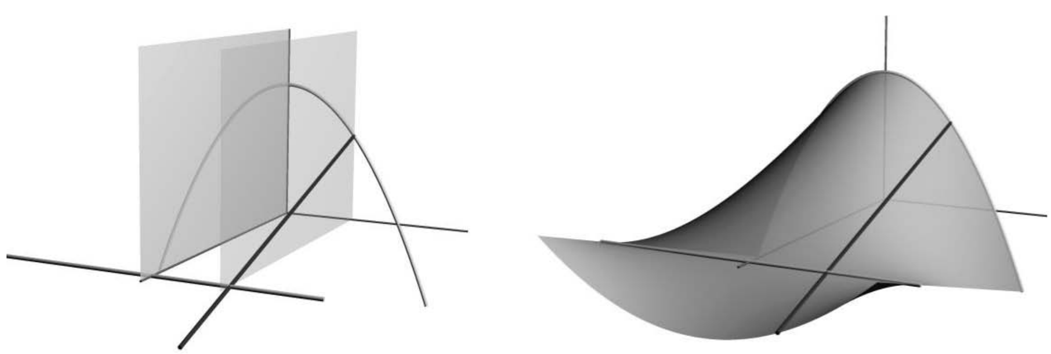


Figure 7: Illustration of the generation of ruled surfaces by a moving straight line. This is the special type of ruled surface which is called a parabolic conoid. Straight lines on the surface are parallel with the depicted plane and intersect parabola and straight line. On the left input elements, on the right the final shape of the conoid.

Surfaces of revolution, helical surfaces, ruled surfaces and other special types of surfaces have been used in art, design, architecture, engineering and so on. We should motivate our students with examples of usages of these surfaces in practical applications. See the following examples of geometric surfaces used in architecture or technical design.

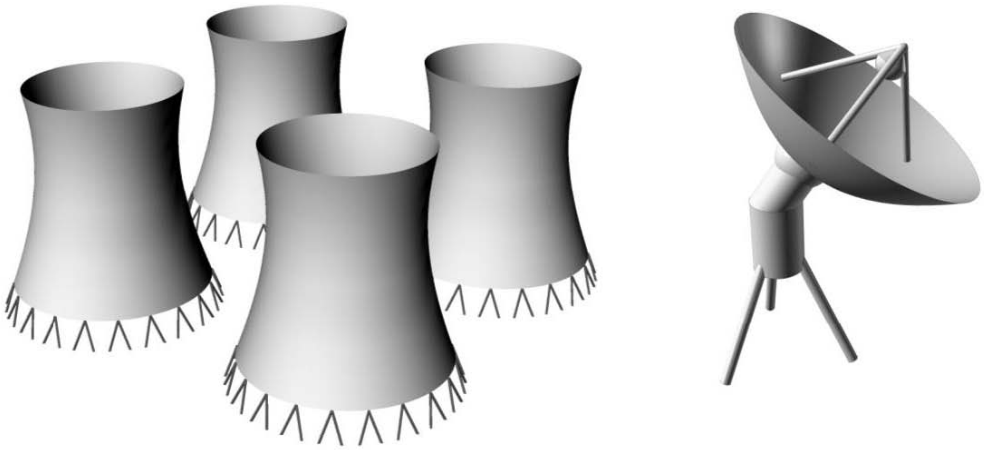


Figure 8: Examples of practical uses of surfaces of revolution; on the left the one-sheet rotational hyperboloids as cooling towers, on the right the rotational paraboloid as radio telescope.

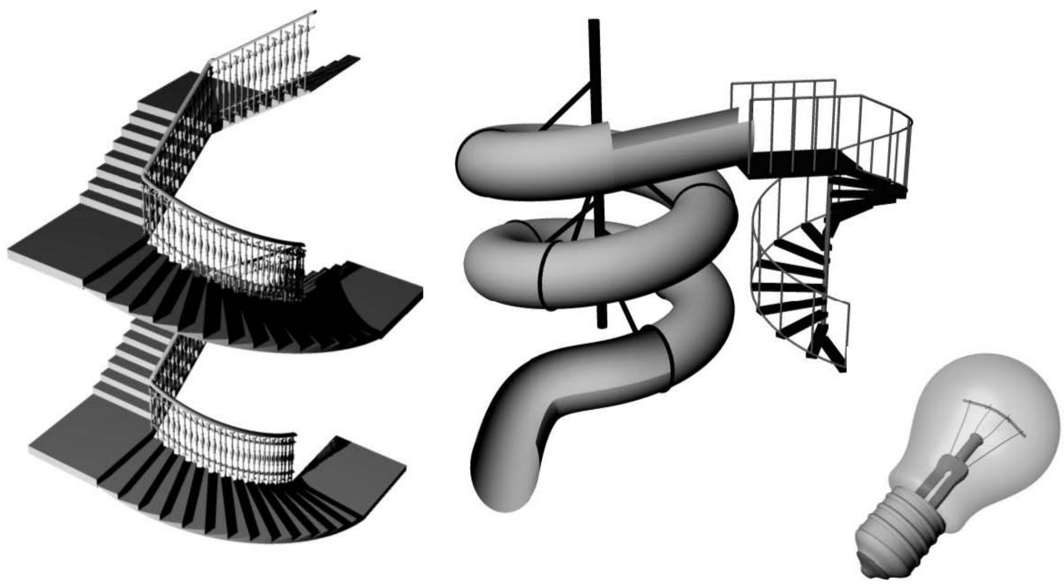


Figure 9: Examples of practical uses of helical surfaces.

Figure 8 shows practical usage of surfaces of revolution – the one-sheet rotational hyperboloid and the rotational paraboloid in the industrial design. Figure 9 demonstrates application of helical surfaces as a spiral staircase – the common helicoid, a water slide – helical surface with circular generator and a bulb with the screw.

Hyperbolic paraboloids are depicted in the figure 10. These surfaces are well established in the area of thin shells in architecture. Positive static properties allow the construction of shells of large span width with relatively small thickness. They are easy-to-use elements for architectural design and offer many design possibilities.

These pictures were also made in Rhinoceros.

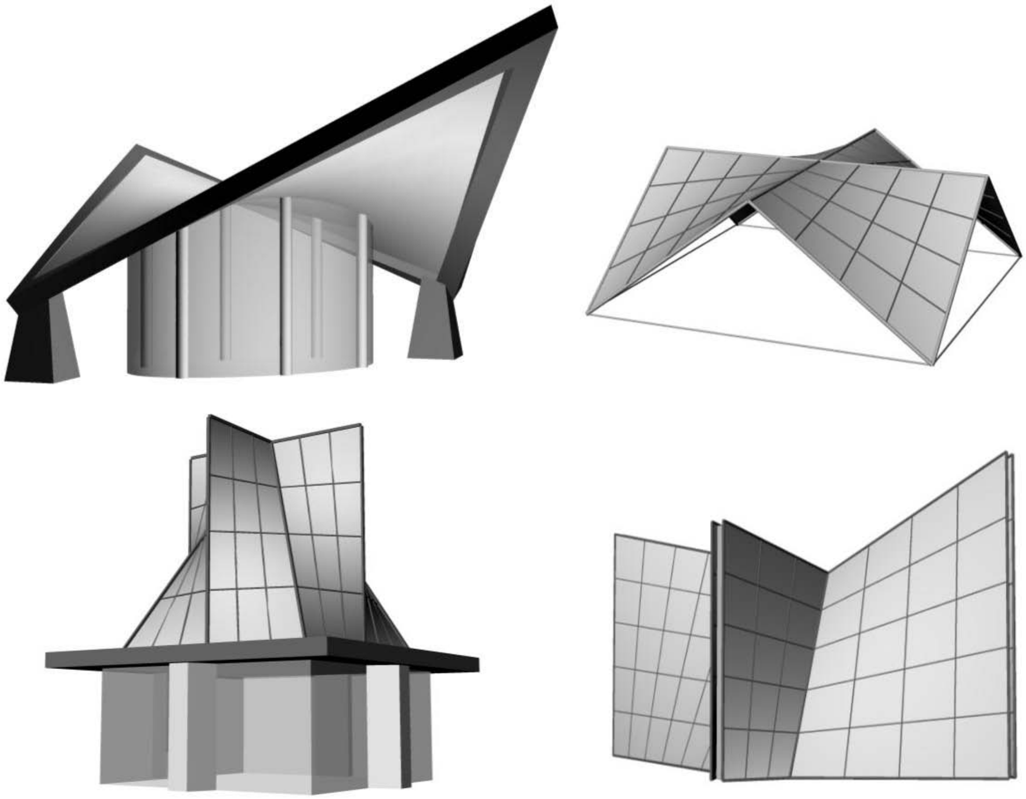
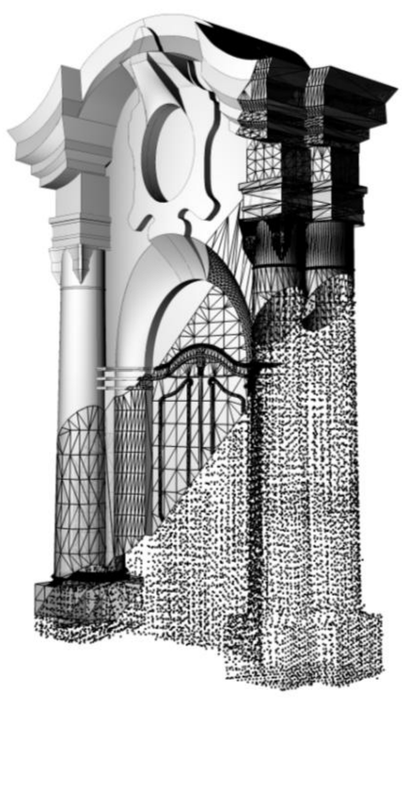


Figure 10: The hyperbolic paraboloids in use by architectural design.



GEOMETRY IN DIGITAL SURFACE RECONSTRUCTION FROM POINT CLOUDS

In this section we explain the subject and scope of my PhD studies. Several results from my research achieved during my PhD studies will be discussed because the main area of my interest is applied and computational geometry. I use three dimensional geometry and geometric algorithms to solving very popular problem of digital surface reconstruction from the point cloud. So we can demonstrate the usage of geometry in very modern practical area. This approach can motivate students for studying geometry, too. We can also show that it is a great advantage to have knowledge of classical geometry in area such a surface reconstruction because I use it in my research all the time.

Let us explain briefly the problem area of the surface reconstruction. We explore the steps which are necessary to convert a physical model or some real object into a computer model. Of course, good geometric and a solid understanding of the procedure are essential to get excellent result. Digital documentation brings the possibility to manipulate with the objects in mathematical and modeling computer software. The digital documentation of real objects is important in many branches. For example, in the architectural engineering it can help with reconstruction and documentation of historical buildings and sculptures with 3D scanners or restoring of monuments. The digital reconstruction is applied in many scientific and engineering applications.

The input is a finite set of points in the three dimensional space, we know only 3D coordinates. The input set is called point cloud in computer graphics. Three dimensional scanners are used to produce measurement data from a real three dimensional object. At first, before scanning, we have to think about what we consider as relevant data to be captured. There are many aspects of an object – its surface geometry, its appearance, its materiality and its geometric features. For example, a sculptural object might be best captured with a scattered set of key edges in space rather than with undifferentiated point cloud from a three-dimensional scanner. Figure 11 shows an example of digital reconstruction. You can see the input set of points, triangle mesh and the final computer model.

The measurement data can consist of a large number of points. Real data may contain over million points. Ideally, these data are precise coordinates of points on the surface of the object. But in real applications there will be measurement errors, we have to deal with. Only the regions of the surface of the real object directly visible from one position of the scanner will be captured. So that a single scan usually contains only measurement data for a part of the real object. We have to produce a number of scans from various positions of the three dimensional scanner. Each scan produces a point cloud in different coordinate system. All of these obtained point clouds have to be merged into a single point cloud represented in the same coordinate system. This procedure is called registration, see figure 12. In the merged point cloud there may be redundant data, some points are useless, don't contain any new or important information or some points are very close to one another. For that reason these redundant data points will be removed. There exist several removal criteria which depend on the underlying application, more detailed information are in (Iske, 2004).

In the subsequent polygon phase, a triangle mesh is computed that approximates the given data points. This procedure is very difficult. It doesn't exist any general solving method. In the polygon phase we obtain a first surface representation of the object. Several known algorithms for computing triangle mesh are for example alpha-shapes, crust algorithm, cocone algorithm which are based on spatial subdivision (on the dividing of the three dimensional space). It means that the circumscribed box of the input set of points is divided into disjoint cells – e.g. tetrahedrization, we obtain a tetrahedral mesh. Then we find those parts of mesh which are connected with the surface (Edelsbrunner et al., 2001).

The final shape phase isn't necessary for pure visualization but it will be crucial for architecture. We have to convert the triangle mesh into a CAD representation of an object that is appropriate for further processing. This phase includes edge and feature line detection and decomposition into parts of different nature and geometry – for example planar parts, cylindrical patches, conical patches, freeform patches. This process is called segmentation. Then we have to approximate the data regions using surfaces of the correct type which we identified in the segmentation. For example, a region identified as being planar in the segmentation phase will be approximated by part of a plane. Computing such an approximation plane is simple task. This process is known as surface fitting. More detailed information can be found in (Pottman et al., 2007).

This is very brief introduction to the surface reconstruction. We implement known algorithms for surface reconstruction and develop new methods of reconstruction based on the sequential evolution (Surynková, 2010), see figure 13. Every step of the digital reconstruction is illustrated. We use three dimensional geometry, geometric algorithms and known mathematical methods for example gradient descent optimization. These processes need to be depicted and modeled. You can see examples of the illustrations made in Rhinoceros.

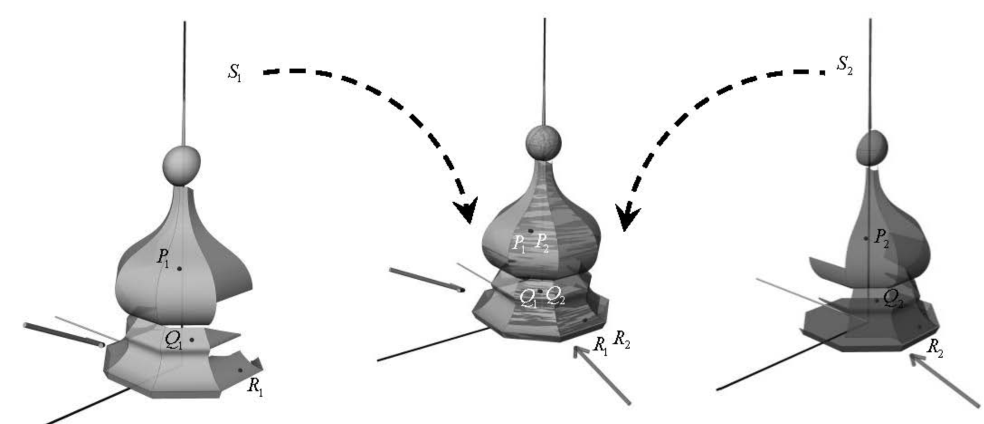


Figure 12: The illustration of registration.

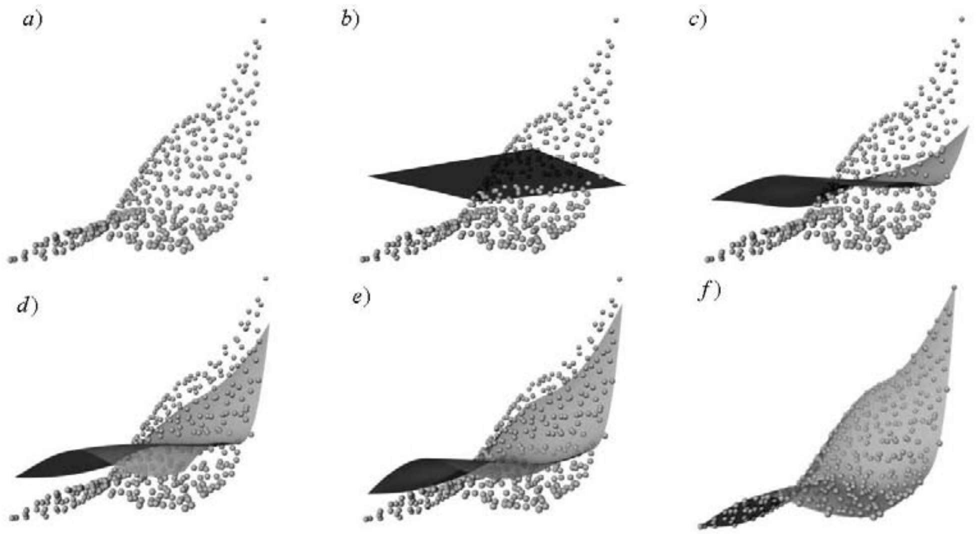


Figure 13: The illustration of sequential evolution – a) the input point cloud in the space, b) initial position and shape of the surface and the input set of points, c) – e) surface evolution in time, f) the final surface.

CONCLUSION AND FUTURE WORK

We discussed possible approaches how to increase the interest in studying geometry by using computers. The usage of 3D modeling we demonstrated on some concrete examples from the field of descriptive geometry mainly geometry of surfaces and computational geometry. If we can work with 3D model of the object in modeling software and move with it we understand more its properties. We also demonstrated the usage of geometry in very modern practical area – digital surface reconstruction from the point cloud. I use for solution this problem three dimensional geometry and geometric algorithms. This approach can motivate students for studying geometry, too.

The responses from students to using mathematical and modeling computer software (for example Rhinoceros) in teaching geometry are very positive. Students are satisfied because the computer software is very motivational and attractive for them. Classical and descriptive geometry are more understandable and geometry in general becomes modern discipline.

We also have web pages with database of geometric tasks, images and 3D models (Surynková, 2011) – the outputs of various software which are suitable teaching aid can be used in publications and also for e-learning not only for our students. In future work we will focus on further methods which can improve the teaching process. We plan to extend our gallery of 3D models and geometric tasks.

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